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ARTICLE

Assessing and hedging the impact of longevity risk for countries with limited data

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Abstract
Almost all literature on the impact of longevity concludes that its impact is huge and can collapse any life or pension company if steps are not taken to address it. Yet, life companies in most developing countries do not account for longevity risk. This is a result of the lack of suitable mortality data needed for such valuation. In this work, we have proposed a generalized method of assessing the impact of longevity risk when mortality data is scarce and shown theoretically that our earlier proposed method is a particular case. This means that this method can be used by not just pension companies but all life companies. The method is based on our earlier proposed model which shows that there is a nearly linear relationship between annuitant's hazard function and their mortality at higher ages (post-retirement age) which permits approximating post-retirement mortality data with the Gompertz model. The work also considers how such a risk could be managed under the assumption of limited mortality data, and shows that a range of life products whose expected return depends on the distribution of individual lifetimes could be used to hedge such a risk. Specifically, we showed how a whole life annuity product could be used to hedge such a risk.

Keywords: Hedging; Longevity Risk; Interest Rate; Limited Data; Mortality..

1. Introduction

The impact of longevity risk has been widely discussed in the literature and they all seem to show longevity risk can collapse any life company if measures are not taken to address it. Most of these conclusions are based on experiences from developed countries with very few addressing the problem from developing countries' perspective. Even the literature that considers longevity risk from developing countries perspective largely concerns countries in Europe, Asia, and American. In the case of Africa, not much work has been done as far as longevity risk is concerned even though the region experienced the highest growth in life expectancy according to the 2017 UN report (2017). This is mainly due to the lack of suitable mortality data and models that are needed to assess the impact of longevity risk. As a result, most life companies in these countries rely on life tables from other countries which may not suit their particular need. It is against this background that Assabil and McLeish (2020) proposed a method of assessing longevity risk in situations where mortality data is severely scarce. Using mortality data from Ghana, the work showed that longevity risk is present in Ghana and its impact could be high if steps are not taken to address it.

One limitation of the work is that their method only applies to pensions. Also, even though the work concluded that longevity risk is present in Ghana and its impact could potentially be high, they did not show how such a risk could be managed under the assumption of limited data.

Generally, several ways to manage longevity risk have been proposed in literature but they all seem to be silent on the situation where there is limited mortality data. For example, Blake et al. (2014) suggested hedging longevity risk on the capital market with longevity link products such as longevity bond or survivor bond. More specifically, they suggested the government issued a longevity bond as a way of managing longevity risk. This is also supported by Muralidhar (2018) who suggested that the government should create Longevity- Indexed Variable Expiration bonds. These bonds, targeted to individuals (and institutions) would pay income-only, and start paying only after the average life expectancy of society. This method of managing longevity risk with longevity bonds requires good and reliable mortality data which are not available in most developing countries. Roy (2012), on the other hand, suggested the use of annuities as a way of managing longevity risk. Annuities are of various forms and depending on individual needs, one could purchase an annuity to meet that need. Thus, a pension company could purchase an annuity whose payment varies according to an individual's life expectancy to manage their longevity risk. This again requires good data on annuitants mortality which is not available in most developing countries.

Reinsurance offers another way of managing longevity risk. With the availability of reinsurance, insurance companies have a bigger capacity to accept bigger risks including Longevity risk. This will also require good mortality data on clients so that reinsurance could be priced accurately.

Rather than using reinsurance, Lorson, and Wagner (2012) suggested the use of securitization as a means of managing longevity risk. According to them, securitization also transfers risk to the third party and can be used as a substitute for reinsurance. Like reinsurance, this also requires good data. Siu-Hang Li and Luo (2012) proposed a longevity hedging strategy that is based on matching mortality rate sensitivities as a way of managing longevity risk. Specifically, they introduce a measure, called key q-duration, which allowed them to estimate the price sensitivity of a life-contingent liability to each portion of the underlying mortality curve. And there are few other methods of managing longevity risk but they all require good and reliable mortality data which is non-existing in most developing countries especially those on the African continent. In the absence of such data, it is almost impossible to use any of the above methods to manage longevity risk. Thus, most developing countries rely on mortality data and models from advanced countries which may not meet their specific objective.

To deal with the data problem faced by these countries, Roy (2012) suggested that retirement age should be tied around each countries life expectancy. But again, most of these countries already suffer from a high unemployment rate, and increasing the retirement age may further worsen the unemployment problem in these countries. An alternative is to find a way to deal with the data problem in these countries. This work, therefore, seeks to achieve two main objectives: first, to propose a generalized method of assessing the impact of longevity risk when there is a severe lack of mortality data to permit the use of standard models and also, to show how such a risk could be managed in the context of developing countries where mortality data is scarce.

2. The Model

In earlier work, Assabil and McLeish (2020) considered longevity risk in the context of limited data. They developed a model for assessing the impact of longevity risk in cases where there is a severe shortage of mortality data. Using the 2010 Ghanaian mortality data, they showed that longevity risk is present in Ghana and its impact can cripple any pension company in the country if steps are not taken to address it. The model is a simple approximation to an annuity which is particularly useful when mortality data is scarce or highly reliable data is not available as is the case in most developing countries. The model can be stated as

$$A_\nu \simeq \kappa(\widetilde{\Lambda}(c))^{\tilde{\tilde{r}}\kappa} e^{\widetilde{\Lambda}(c)} \Gamma(-\tilde{r}, \widetilde{\Lambda}(c)) \tag{1}$$

and its derivatives with respect to changes in mortality by

$$\frac{\frac{\partial}{\partial \beta_0} A_\nu}{A_\nu} \simeq \tilde{\kappa}\kappa + \widetilde{\Lambda}(c) - \frac{\kappa}{A_v} \tag{2}$$

where κ is such that, approximately,

$$\frac{1}{\kappa} \simeq \frac{\mu(x) - \mu(c)}{\Lambda(x) - \Lambda(c)} \text{ for } x > c \tag{3}$$

and $\widetilde{\Lambda}(x) = \kappa e^{\beta_0 + \frac{x}{\kappa}}$. We call this preposition 1. Preposition 1 was only applied to pensions and this section seeks to extend the model to include all annuities whose expected return depends on the distribution of individual lifetimes.

Suppose that the current price of a contract is a function g of the lifespan X of a random individual in a population of elderly (those comparable in age to those to whom we owe pensions). Then the current expected value of this contract may be written as

$$E[g(X) \mid X \geq a] \tag{4}$$

where the expected value is taken assuming that mortality at higher ages could be approximated by the Gompterz model, valid for reasonably large a. Then the following proposition holds: Preposition 2 Assume $\Lambda(x) = \kappa e^{\beta_0 + \frac{x}{\kappa}}$ is the Gompertz approximation to the cumulative mortality, valid for $x \geq a$ and $\Lambda^{-1}(y) = \kappa \ln\left(\frac{y}{\kappa}\right) - \kappa\beta_0$. If the current value of the contract is given by an expected value

$$V = E[g(X) \mid X \geq a] = e^{\Lambda(a)} \int_{\Lambda(a)}^{\infty} g\left(\Lambda^{-1}(y)\right) e^{-y} dy$$

then its logarithmic derivative with respect to longevity changes is given by

$$\frac{\frac{\partial}{\partial \beta_0} V}{V} = 1 + \Lambda(a) - \frac{E[g(X)\Lambda(X) \mid X \geq a]}{V} = 1 + \Lambda(a) - \frac{e^{\Lambda(a)}}{V} \int_{\Lambda(a)}^{\infty} g\left(\Lambda^{-1}(y)\right) y e^{-y} dy. \tag{5}$$

Proof

In our earlier paper, Assabil and McLeish (2020), we justified empirically the approximation $\Lambda(x) = \kappa e^{\beta_0 + \frac{x}{\kappa}} + c_1$ for cumulative mortality and large values of age x and assumed without loss of generality that $c_1 = 0$ and $\mu(x) = \Lambda'(x) = e^{\beta_0 + \frac{x}{\alpha}}$. The conditional probability density function of X given $X \geq a$ is given by

$$f(x \mid X > a) = \mu(x) e^{-\Lambda(x)}, x > a \tag{6}$$

And the score function is

$$S(x) = \frac{\partial}{\partial \beta_0} \ln(f(x \mid X \geq a)) = 1 + \Lambda(a) - \Lambda(x), x > a$$

Moreover, in general, the derivative with respect to a parameter of the expected value of a function is the covariance between the function and the score (see e.g. McLeish and Small,(1988).

$$\frac{\partial}{\partial \beta_0} \mathrm{E}[\mathrm{g}(\mathrm{X}) \mid \mathrm{X} \geq a] = E[g(X)S(X) \mid \mathrm{X} \geq a]$$

from which we have

$$\frac{\partial}{\partial\beta_0}\mathrm{E}[\mathrm{g}(\mathrm{X}) \mid \mathrm{X} \geq a] = E[g(X)(1 + \Lambda(a) - \Lambda(X)) \mid \mathrm{X} \geq a]$$

$$= (1 + \Lambda(a))V - E[g(X)\Lambda(X) \mid \mathrm{X} \geq a$$

$$= (1 + \Lambda(a))V - \int_a^\infty g(x)\Lambda(x)\mu(x)e^{\Lambda(a)-\Lambda(x)}dx$$

$$= (1 + \Lambda(a))V - e^{\Lambda(a)}\int_{\Lambda(a)}^\infty g\left(\kappa\ln\left(\frac{\gamma}{\kappa}\right) - \kappa\beta_0\right)\gamma e^{-\gamma}d\gamma$$

As an example, consider a life insurance policy beneficiary who receives a benefit of 1 on the death of an individual currently at age a. In this case the present value is given by

$$g(x) = e^{-r(x-a)} \text{ for } x > a$$

and the expected value is given as

$$D_b(r, a) = E\left[e^{-r(x-a)} \mid \mathrm{X} \geq a\right. = (\Lambda(\mathrm{a}))^{r\kappa}e^{\Lambda(a)}\Gamma(1 - r\kappa, \Lambda(a)) \tag{7}$$

Also, the derivative $\frac{\partial}{\partial\beta_0}D_b$ is given as

$$\frac{\frac{\partial}{\partial\beta_0}D_b(\mathrm{r}, \mathrm{a})}{D_b(\mathrm{r}, \mathrm{a})} = 1 + \Lambda(a) - \frac{E[g(X)\Lambda(X) \mid \mathrm{X} \geq a}{D_b(\mathrm{r}, \mathrm{a})} = 1 + \Lambda(a) - \frac{\Gamma(2 - r\kappa, \Lambda(a))}{D_b(\mathrm{r}, \mathrm{a})}$$

This can be simplified somewhat to give

$$\Gamma(2 - r\kappa, \Lambda(a)) = (\Lambda(a))^{1-r\kappa}e^{-\Lambda(a)} + (1 - r\kappa)\Gamma(1 - r\kappa), \Lambda(a)\Big)$$

Then, on substitution and some simplification,

$$\frac{\frac{\partial}{\partial\beta_0}D_b(r, a)}{D_b(r, a)} = r\kappa + \Lambda(a) - \frac{\Lambda(a)}{D_b(\mathrm{r}, \mathrm{a})} > 0 \text{ if } r > 0$$

This derivative is positive since for positive interest rates, a decrease in mortality results in a deferral in the payment of the death benefit and consequent reduction in its present value.

2.1 Particular case of our model

Here, we show that our earlier proposed method for assessing the impact of longevity risk in pensions (which we call Preposition 1) is a particular case of our model. Given that

$$g(X) = \frac{1}{r}\left(1 - e^{-r(X-c)}\right)A_v = \frac{1}{r}E\left[\left(1 - e^{-r(X-c)}\right) \mid X > c\right] = \frac{1}{r} - \frac{1}{r}E\left[e^{-r(X-c)} \mid X > c\right]$$

And using the integral

$$E\left[e^{-r(x-a)}\Lambda^p(X) \mid X \geq a\right] = \kappa(\Lambda(a))^{r\kappa}e^{\Lambda(a)}\Gamma(p + 1 - r\kappa, \Lambda(a))A_v = \frac{1}{r} - \frac{1}{r}(\Lambda(c))^{r\kappa}e^{\Lambda(c)}\Gamma(1 - r\kappa, \Lambda(c))$$

To relate this to our previous formula for A_v, we use the substitution

$$\Gamma(1 - r\kappa, \Lambda(c)) = (\Lambda(c))^{-r\kappa}e^{-\Lambda(c)} - r\kappa\Gamma(-r\kappa, \Lambda(c))$$

to obtain,

$$A_\nu = \frac{1}{r} - \frac{1}{r}(\Lambda(c))^{r\kappa} e^{\Lambda(c)} \left[(\Lambda(c))^{-r\kappa} e^{-\Lambda(c)} - r\kappa\Gamma(-\text{r»}, \Lambda(\text{c}))\right]$$

$$= \frac{1}{r} - \frac{1}{r}(\Lambda(c))^{r\kappa} e^{\Lambda(c)} (\Lambda(c))^{-r\kappa} e^{-\Lambda(c)} + \frac{1}{r}(\Lambda(c))^{r\kappa} e^{\Lambda(c)} r\kappa\Gamma(-\text{r»}, \Lambda(\text{c}))$$

$$= \kappa(\Lambda(c))^{r\kappa} e^{\Lambda(c)} \Gamma(-\text{r»}, \Lambda(\text{c}))$$

and this agrees with our previous formula for A_ν. The corresponding derivative is

$$\frac{\partial}{\partial \beta_0} A_\nu = E[g(X)S(X) \mid X > c]$$

$$= (1 + \Lambda(c))A_\nu - E[g(X)\Lambda(X) \mid X > c]$$

$$= (1 + \Lambda(c))A_\nu - \frac{1}{r} E\left[\left(1 - e^{-r(X-c)}\Lambda(X) \mid X > c\right]\right.$$

$$= (1 + \Lambda(c))A_\nu - \frac{1}{r} E[\Lambda(X) \mid X > c] + \frac{1}{r} E\left[\left(1 - e^{-r(X-c)}\Lambda(X) \mid X > c\right]\right.$$

$$= A_\nu + \Lambda(c)A_\nu - \frac{1}{r} - \frac{1}{r}\Lambda(c) + \frac{1}{r} E\left[e^{-r(X-c)}\Lambda(X) \mid X > c\right]$$

Again using the integral

$$E\left[e^{-r(X-a)}\Lambda^p(X) \mid X \geq a\right] = \kappa(\Lambda(a))^{r\kappa} e^{\Lambda(a)} \Gamma(p + 1 - r\kappa, \Lambda(a))$$

and

$$\Gamma(2 - r\kappa, \Lambda(c)) = (\Lambda(c))^{1-r\kappa} e^{-\Lambda(c)} + (1 - r\kappa)\Gamma(1 - r\kappa, \Lambda(c))$$

we obtain

$$E\left[e^{-r(X-c)}\Lambda(X) \mid X > c\right] = (\Lambda(c))^{r\kappa} e^{\Lambda(c)} \Gamma(2 - r\kappa, \Lambda(c))\Big]$$

$$= \Lambda(c) + (1 - r\kappa)(\Lambda(c))^{r\kappa} e^{\Lambda(c)} \Gamma(1 - r\kappa, \Lambda(c))$$

From which we get,

$$\frac{\partial}{\partial \beta_0} A_\nu = A_\nu + \Lambda(c)A_\nu - \frac{1}{r} - \frac{1}{r}\Lambda(c) + \frac{1}{r}\Lambda(c) + \left(\frac{1}{r} - \kappa\right) (\Lambda(c))^{r\kappa} e^{\Lambda(c)} \Gamma(1 - r\kappa, \Lambda(c))$$

However note that from before $A_\nu = \frac{1}{r} - \frac{1}{r}(\Lambda(c))^{r\kappa} e^{\Lambda(c)} \Gamma(1 - r\kappa, \Lambda(c))$

and so $(\Lambda(c))^{r\kappa} e^{\Lambda(c)} \Gamma(1 - r\kappa, \Lambda(c)) = 1 - rA_\nu$. This implies

$$\frac{\partial}{\partial \beta_0} A_\nu = (1 + \Lambda(c))A_\nu - \frac{1}{r}(1 + \Lambda(c)) + \frac{1}{r}\Lambda(c) + \left(\frac{1}{r} - \kappa\right) (1 - rA_\nu)$$

$$= A_\nu + \Lambda(c)A_\nu - \frac{1}{r} - \frac{1}{r}\Lambda(c) + \frac{1}{r}\Lambda(c) + \frac{1}{r} - \kappa - A_\nu + r\kappa A_\nu = (r\kappa + \Lambda(c))A_\nu - \kappa$$

which agrees with the formula in Proposition 1.

Hedging

Suppose an insurance company is short a single annuity valued at $-A_v(\tilde{r}, c)$ that we wish to hedge against longevity risk. Since as mortality rate decreases the value of this annuity increases and $-A_v(\tilde{r}, c)$ decreases, a whole life insurance policy might be used as a hedging instrument since its present value increases with decreasing mortality. In fact there may be a range of products whose expected return depends on the distribution of lifetimes X such as products marketed to seniors or the retired. Suppose the expected return per individual in the population from such a product is given by (4) and a death benefit of δ is received on an individual currently at age a. Then the present value may be determined by (4) with

$$g(x) = \delta e^{-r(x-a)} \text{ for } x > a$$

and the expected value is given by (7). Now consider the vendor of a whole life insurance policy that receives premiums ρ unit per unit time after the individual is of age a and then pays out a death benefit of δ. This can also be written in the form (4) where

$$g(X) = \rho \int_a^X e^{-r(z-a)} dz - \delta e^{-r(X-a)} = \frac{\rho}{r} - \left(\frac{\rho}{r} + \delta\right) e^{-r(X-a)}$$

Such a policy is the sum of the premium annuity and the (negative) death benefit, and its present value, given by (1), is

$$L_a = \frac{\rho}{r} - \left(\frac{\rho}{r} + \delta\right) D_b(r, a) \tag{8}$$

with D_b given by (7). In the special case $\delta = 0$ and $\rho = 1$ we obtain the value $A_v\ (r, a) = \frac{1}{r}\left(1 - D_b(r, a)\right)$ of a pension annuity to the recipient of the pension. As mortality decreases, the present value $L_a(r)$ of the insurance policy increases since the number and value of premiums goes up and payment of the death benefit is later so the discounted expected death benefit smaller. We also have the sensitivity of this policy with respect to changes in mortality as

$$\frac{\partial}{\partial \beta_0} L_a = -\left(\frac{\rho}{r} + \delta\right) \frac{\partial}{\partial \beta_0} D_b \tag{9}$$

where $\frac{\partial}{\partial \beta_0} D_b$ is given at (6). Note that we assume interest rate, $r > 0$, and since $\frac{\partial}{\partial \beta_0} D_b > 0$, it follows that $\frac{\partial}{\partial \beta_0} L_a < 0$, or that the value of the Life insurance contract to the insurer is decreasing function of mortality. In this respect it behaves similarly as the pension annuity to the recipient of the pension. In order that such a contract be a useful hedge to a short position in a pension annuity with value $-A_v(\tilde{r}, c)$ we require a long position in a contract that is a decreasing function of mortality, such as a portfolio of life insurance contracts above. In general, if we assume that an insurance company has a portfolio of whole life contracts with different death benefits δ_i and premium ρ_i and different current ages of insurance holders a_i. And it has w_i of each of these and allow for fractional (or negative) values of w_i. Then

$$g(X) = \rho \int_a^X e^{-r(z-a)} dz - \delta e^{-r(X-a)} = \sum_i w_i \frac{\rho_i}{r} - \sum_i w_i \left(\frac{\rho_i}{r} + \delta_i\right) e^{-r(X-a_i)}$$

and, as above, the current value of such a portfolio is

$$L_a = \sum_i w_i \frac{\rho_i}{r} - \sum_i w_i \frac{\rho_i}{r} \left(\frac{\rho_i}{r} + \delta_i\right) D_b\,(r, a_i)$$

and its derivative with respect to mortality is

$$\frac{\partial}{\partial \beta_0} L_a = \sum_i w_i \left(\frac{\rho_i}{r} + \delta_i \right) \frac{\partial}{\partial \beta_0} D_b \left(r, a_i \right)$$

If the company has a short position in a pension annuity whose value the company wishes to hedge, then the company is able purchase long or short positions in life insurance on a sufficiently large portfolio of individuals to allow for hedging longevity risk since there are many other financial products whose value depends on longevity, including the securities of publicly traded life insurance companies themselves.

3. Conclusion

Assessing the impact of longevity risk in most developing countries has largely been ignored due to the lack of suitable mortality data to carry out such an assessment. As a result most life companies in these countries rely on data and models from other countries which may not suit their particular need. It is against this background that Assabil and Don (2020) proposed a method of assessing longevity risk in pensions when there is a severe lack of mortality data. In this work we have proposed a generalized form of the model to include all annuity products whose expected return depends on the distribution of individual lifetimes and showed that our earlier proposed method is a particular case. The model is based on our earlier proposed method in which we showed that there is a nearly linear relationship between annuitant's hazard function and their mortality at higher ages (post-retirement age) which permit approximating the post-retirement period with the Gompertz model. The work also considers how longevity risk can be managed in the context of limited mortality data. It shows that since a decrease in mortality rate results in an increase in our annuity value (and $A_v(\tilde{r}, c)$ decreases), a range of products whose expected return depends on the distribution of individual lifetimes could be used to hedge such a risk. As an illustration, we use whole life annuity product. With this method, pension and other life companies will not only be able to assess the longevity risk they face but also will be able to hedge it using suitable products.

Biographical notes

Dr. Samuel Assabil is a lecturer at the Department of statistics, University of Cape Coast. I have consulted with various companies in the financial services industry and also served on a number of committees (Chairman on Ex-Gratia committee, Conditions of service review committee etc) at the university. My research interest is in the application of statistical models to financial data as well as mortality modelling with application to pension and insurance. I also have interest in health modelling and past work include Modeling mortality with covariate, Forecasting maternal mortality with modified Gompertz model, statistical approach to automated blood pressure and so on.

Dr. Francis Eyiah-Bediako is a senior lecturer and the current head of Department (statistics), University of Cape Coast. I have served on a number of committees and as a former Hall Master at Ogua Hall. My research interest is in the application of multivariate techniques and principal component analysis. I also research on application of statistical techniques to healthcare and past research include Modeling macroeconomic variable using principal component analysis, Time series analysis and forecasting

Conflict of interest

The authors declare no conflict of interest.

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Appendix

Some useful integrals

We provide commonly occurring integrals assuming a Gompertz approximation, of $\Lambda(x) = \kappa e \beta_0 + \kappa x$ for sufficiently large x (when $x \geq a$) and use the corresponding distribution with c.d.f. given by (3) to provide the following integrals: for p = 0, 1, 2, . . . 1 $E\left[e^{-r(x-a)}\Lambda^p(X) \mid X \geq a\right] = \kappa(\Lambda(a))^{r\kappa} e^{\Lambda(a)}\Gamma(p + 1 - r\kappa, \Lambda(a))$ 11 African Review of Economics and Finance 2 In the special case $r = 0$ we obtain

$$E\left[\Lambda^p(X) \mid X \geq a\right] = \kappa e^{\Lambda(a)}\Gamma(p + 1, \Lambda(a))$$

and if, in addition, $p = 1$ we obtain

$$E[\Lambda(X) \mid X \geq a] = \kappa e^{\Lambda(a)}\Gamma(2, \Lambda(a)) = \kappa(\Lambda(a) + 1)$$

3 In the special case $p = 0$ we obtain

$$E\left[e^{-r(x-a)} \mid X \geq a\right] = \kappa(\Lambda(a))^{r\kappa} e^{\Lambda(a)}\Gamma(1 - r\kappa, \Lambda(a))$$

4 Using integration by parts, note that

$$\Gamma(p+1,\gamma) = \int_{\gamma}^{\infty} x^{p} e^{-x} dx = -\, x^{p} e^{-x}\Big|_{\gamma}^{\infty} + \int_{\gamma}^{\infty} p x^{p-1} e^{-x} dx = \gamma^{p} e^{-\gamma} + p\Gamma(p,\gamma).$$

In special case $p = -r\kappa$, $\gamma = \Lambda(c)$ we obtain

$$\Gamma \left(1 - r\kappa, \Lambda(c) = (\Lambda(c))^{-r\kappa} e^{-\Lambda(c)} - r\kappa, \Lambda(c)\right)$$

and when $p = 1 - r\kappa$, $\gamma = \Lambda(c)$ we obtain

$$\Gamma(2 - r\kappa, \Lambda(c)) = (\Lambda(c))^{1-r\kappa}\Big)\, e^{-\Lambda(c)} + (1 - r\kappa)\Gamma(1 - r\kappa, \Lambda(c))$$